BOOK REVIEW

FINITE ELEMENTS IN FLUIDS, VOLUME 6, R. H. Gallagher, G. Carey, J. T. Oden and O. C. Zienkiewicz (eds), Wiley, Chichester, 1986. No. of pages: 376. Price: £42.50/$61.00. ISBN 047190676X.

The 18 chapters of this book represent contributions from 18 research groups applying the finite element method to fluid flow. These papers were selected from those presented at the 5th International Conference on Finite Elements and Flow Problems, which was held at Austin, Texas in January 1984. Books such as these have the potential to be disparate papers that leave important holes in covering the subject. That is not the case here—virtually all important topics are covered and the references are complete and lead the reader to appropriate literature. This book represents a succinct summary of how to apply the finite element method to fluid flow and gives ample examples as well as discussing all important techniques. The editors are congratulated on their selection of papers.

All applications are for either (1) continuity and momentum equations for compressible fluids, or (2) the Navier–Stokes equation, sometimes coupled with the energy equation. The only significant fields not included are turbulent flow and the flow of viscoelastic fluids. For the reader interested in a survey of applications, there are examples of:

(a) two-dimensional, transient and steady state flow of incompressible fluid governed by the Navier–Stokes equation; some of the problems are large (12,000 elements)
(b) three-dimensional, transient flow of incompressible fluid governed by the Navier–Stokes equation
(c) two-dimensional transient flow of compressible fluids governed by the Navier–Stokes equation
(d) extrusion with free surfaces
(e) shallow water equations, modelling flow in estuaries
(f) natural convection in cavities (these are inappropriately called Benard problems—what Benard observed was convection driven by surface tension variations, not density)
(g) two-phase heat transfer in natural convection coupled with the Navier–Stokes equation for crystal growth applications
(h) flow through porous media for partially saturated problems.

Even more important is the wide scope of methods applied to these problems. Virtually every important question is discussed:

(a) Iterative methods that hold promise for extension to three dimensions. These include operator splitting, where the incompressibility constraint is in one set of equations and the non-linear convection terms are in another set. Another method is the conjugate gradient method, including the important question of preconditioning. A third iterative method is decomposition by the use of tensor products so that a time step is composed from a succession of one-dimensional problems.
(b) Graded meshes and moving meshes are considered and meshes are adjusted adaptively. Included are criteria for mesh refinement and how to represent the mesh. Sometimes this is done in terms of the shape of the boundary; other times with stream-tube methods with elements orientated along the streamlines. One paper looks at estimated error and subdivides elements if the error is too big; another paper uses a function depending upon the residual and adds higher order interpolation of some elements if the residual is too big.
(c) Methods for strong convection include streamwise diffusion as well as graded meshes using domain splitting with different time steps in the different regions. One paper uses delimiting flux in the finite element method, and this is patterned after similar finite difference methods. Although the results are only applied to one-dimensional problems, they are truly impressive.
(d) Several papers illustrate the impact of supercomputers and the need to re-examine algorithms.
(e) One paper includes the modern analysis tool of continuation methods, including those for bifurcation.
(f) One application solves a scalar equation which gives the velocity as the gradient of the scalar. The paper shows how to obtain a good approximation for the velocity using the mixed method, including the use of analytic functions to represent singularities.

This volume—the 6th in the series—is a tribute to the value of the finite element analysis. Gone is the paranoia of comparing everything to finite difference methods. In its place is application to virtually every important problem in laminar fluid mechanics, including time-dependent, three-dimensional calculations of the Navier–Stokes equations. The methods are sophisticated, becoming highly tuned and are diverse. This is an important reference in the field of computational fluid mechanics.

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